

- All truths are easy to understand once they are discovered; the point is to discover them.

Galileo Galilei

- We cannot teach people anything; we can only help them discover it within themselves.

Galileo Galilei

“Passion is the genesis of genius.”
— Galileo Galilei

- If you can't explain it simply, you don't understand it well enough.

Albert Einstein

- It's not that I'm so smart, it's just that I stay with problems longer.

Albert Einstein

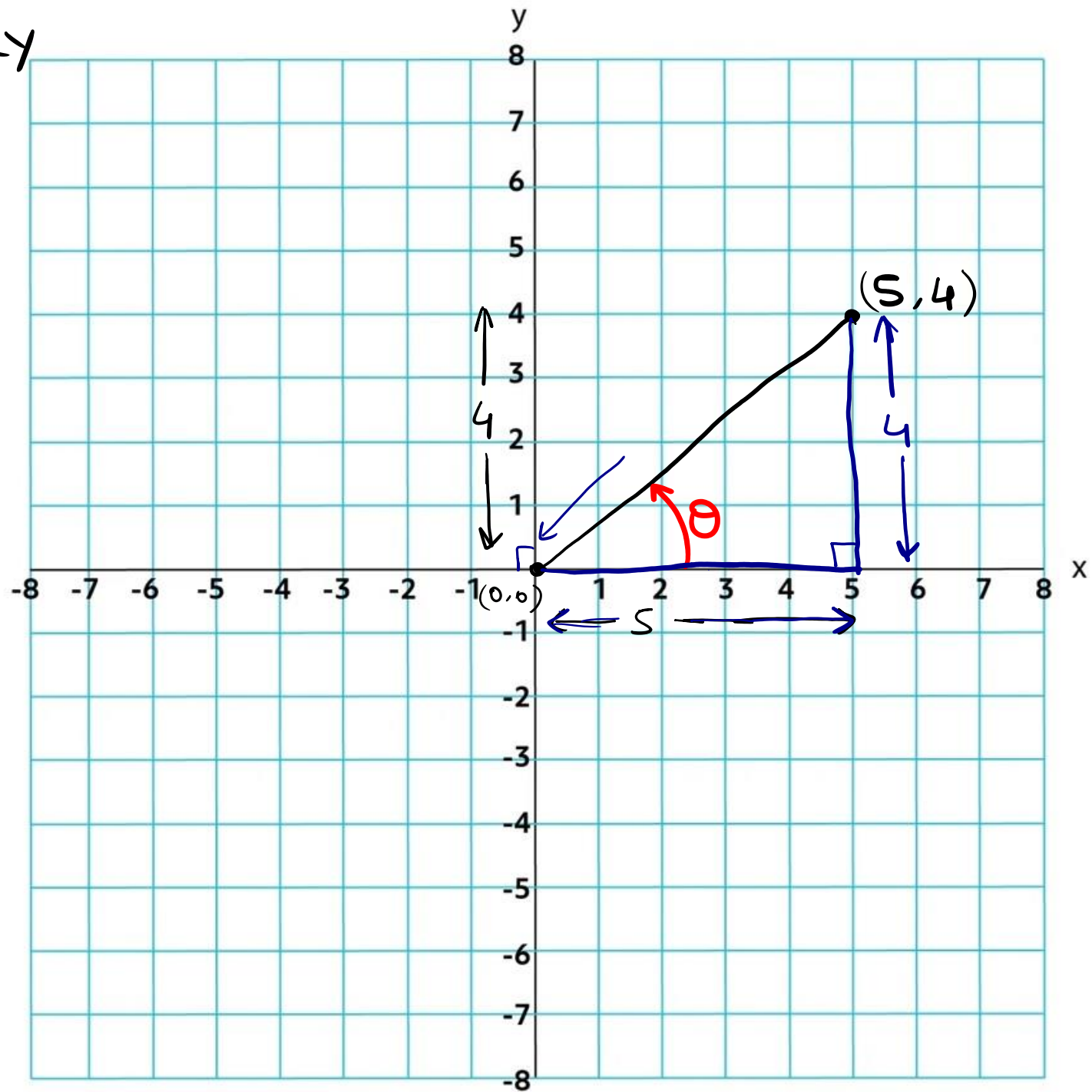
- I have no special talent. I am only passionately curious.

Albert Einstein

- We cannot solve our problems with the same thinking we used when we created them.

Albert Einstein

TRIGONOMETRY



(y axis)
vertical

horizontal
(x axis)

sin θ

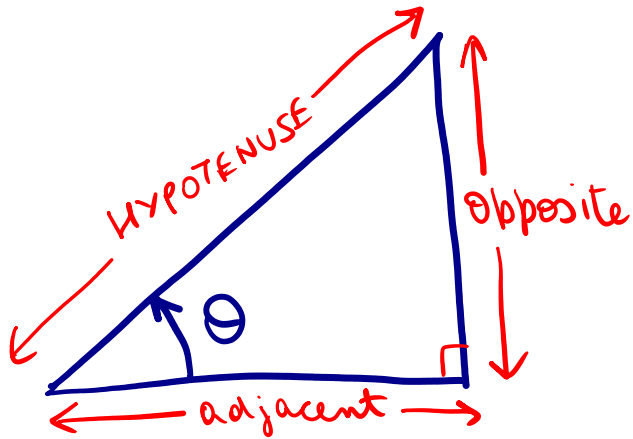
sine

cos θ

cosine

tan θ

tangent



SOH
CAH
TOA

$$\sin \theta = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}$$

$$\cos \theta = \frac{\text{ADJACENT}}{\text{HYPOTENUSE}}$$

$$\tan \theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}} = \frac{\sin \theta}{\cos \theta}$$

$$(\text{Hypotenuse})^2 = (\text{Adjacent})^2 + (\text{Opposite})^2$$

H.W.

Use this relation to derive a relation between $\sin \theta$ & $\cos \theta$.

$$(\sin \theta)^2 + (\cos \theta)^2 = 1 \quad \checkmark$$

$$\theta = 30^\circ$$

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

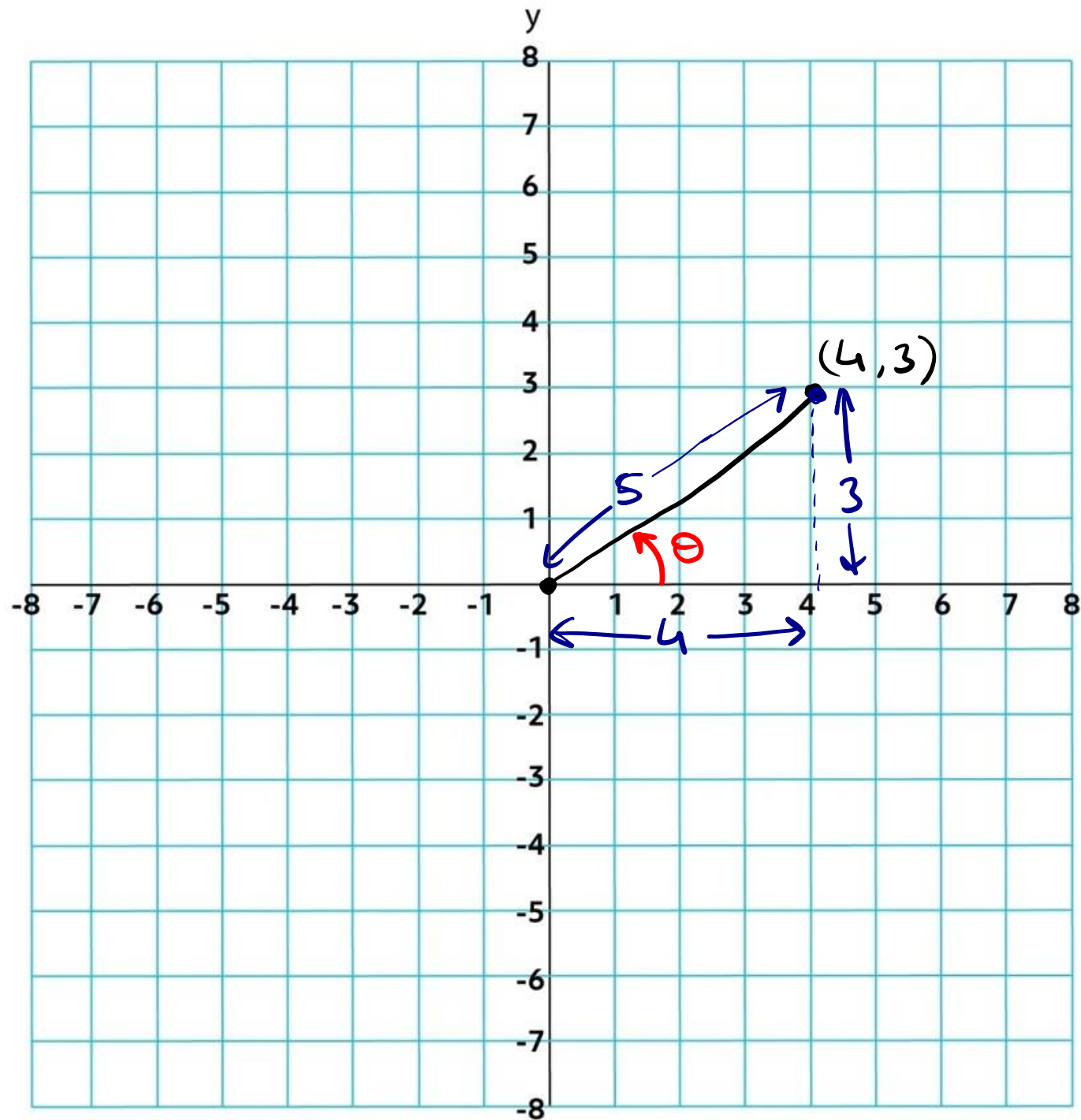
$$\underline{\sin^2 \theta} + \underline{\cos^2 \theta} = \underline{1}$$

$$\underline{\sin^2 30} + \underline{\cos^2 30} = \frac{1}{4} + \frac{3}{4} = 1$$

$$\underline{(\sin \theta)^2} = \sin \theta \times \sin \theta = \boxed{\sin^2 \theta}$$

$$(\sin 30)^2 = \sin^2 30$$

$$\theta \sim 37^\circ$$



$$\sin 53^\circ = 4/5$$

$$\cos 53^\circ = 3/5$$

$$\sin 37^\circ = 3/5$$

$$\cos 37^\circ = 4/5$$

$$\sin \theta = 3/5$$

$$\cos \theta = 4/5$$

$$\theta = ?$$

$$\sin \theta = ?$$

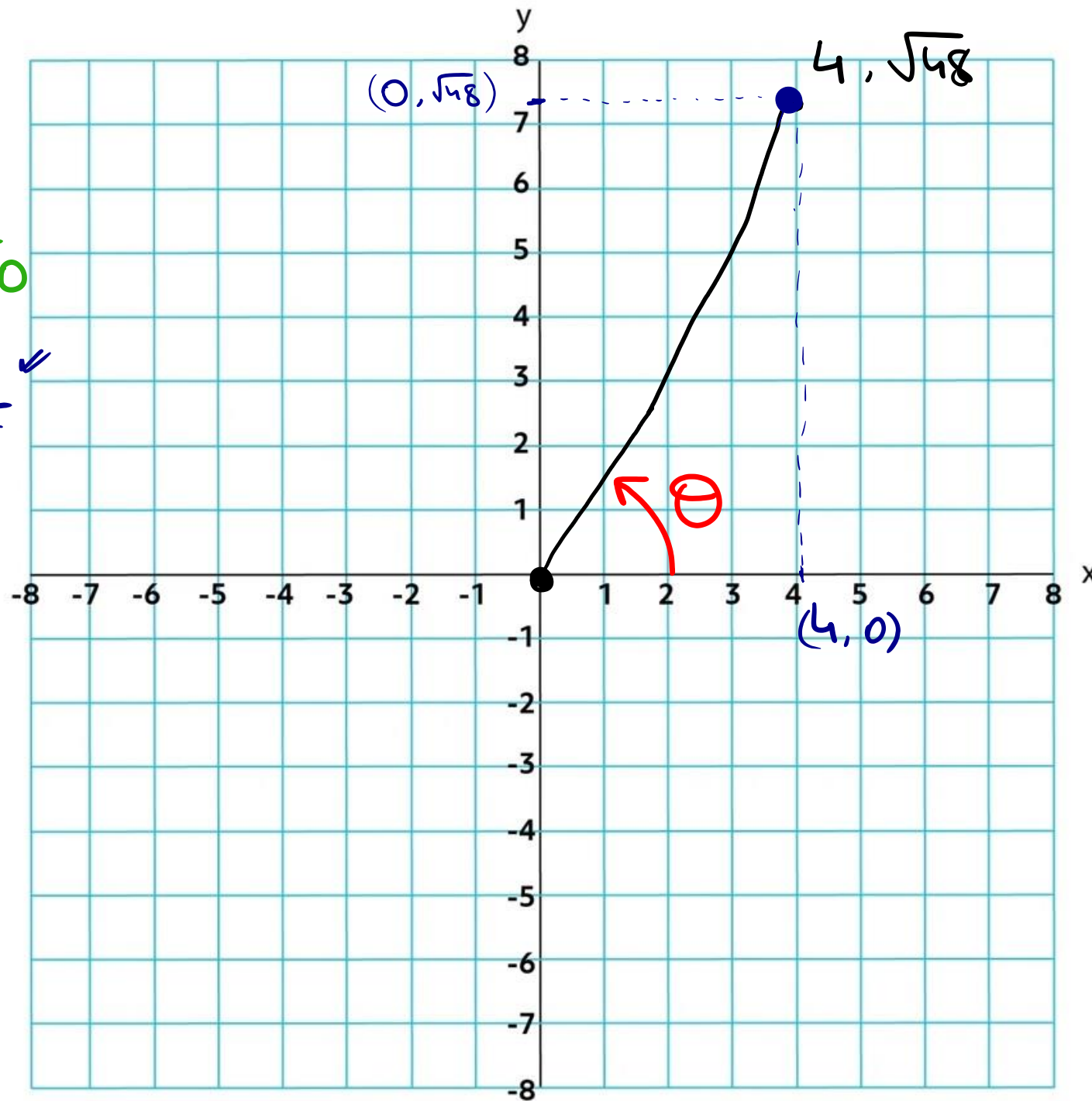
$$\sin \theta = 4 / \sqrt{80}$$

$$\cos \theta = 1/2$$

$$\theta = 60^\circ$$

$$\sin \theta = 1/2$$

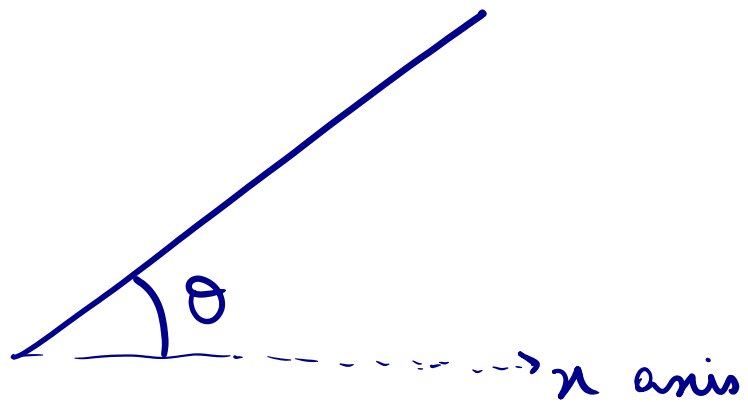
$$\theta = 30^\circ$$



$$\sin \theta = \frac{\text{OPP}}{\text{HYP}} = \frac{\sqrt{48}}{8}$$

$$\begin{aligned} \text{HYP}^2 &= 4^2 + (\sqrt{48})^2 \\ &= 16 + 48 \\ &= 64 \\ \text{HYP} &= \sqrt{64} \\ &= 8 \end{aligned}$$

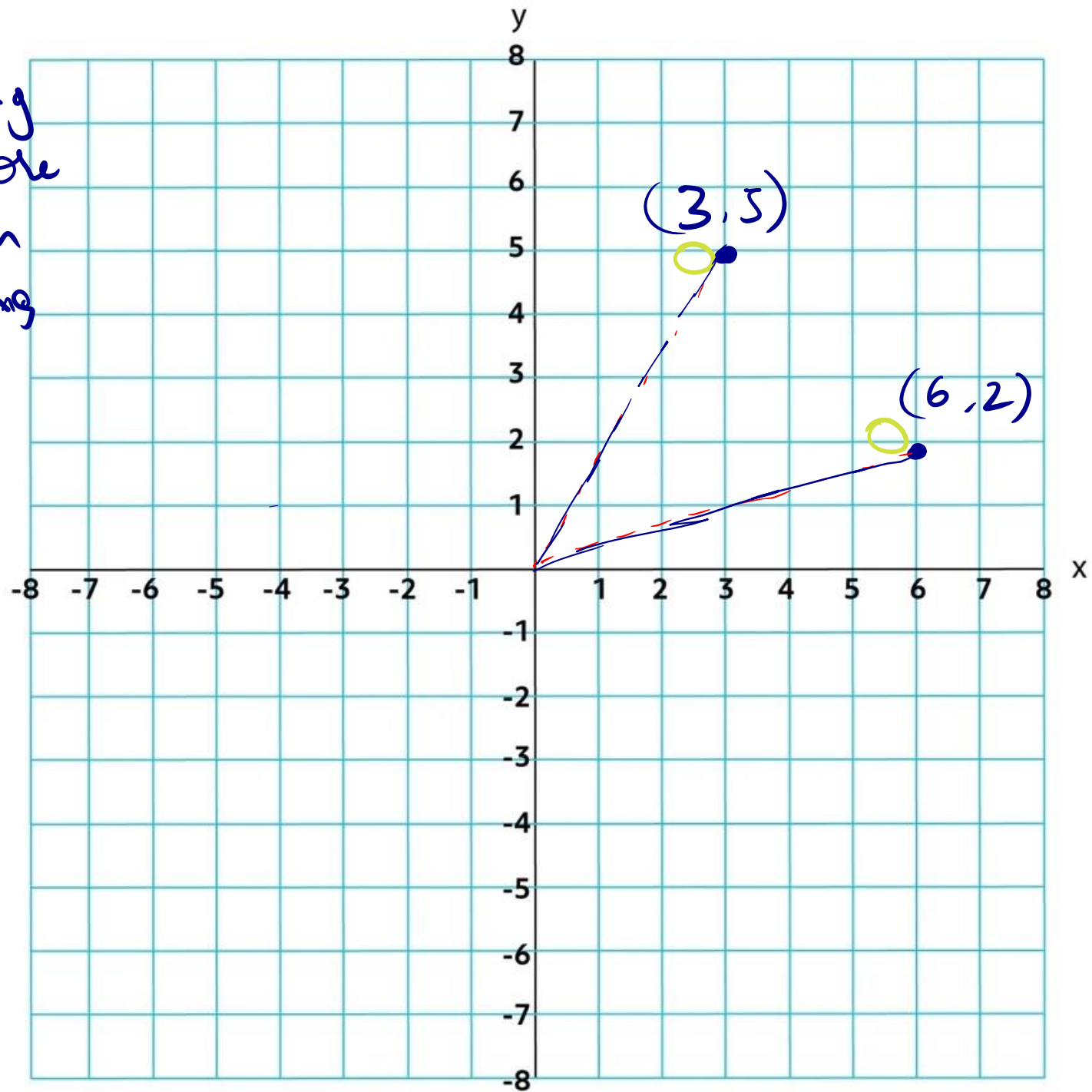
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$



Q - How do I define
SLOPE?

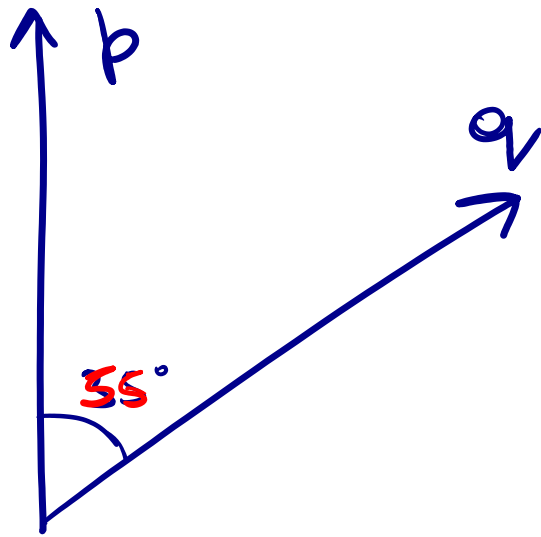
HINTS 1) Mathematically, use
the coordinate axes.

The line joining $(3, 5)$ has more SLOPE than the line joining $(6, 2)$



y
↑
vertical

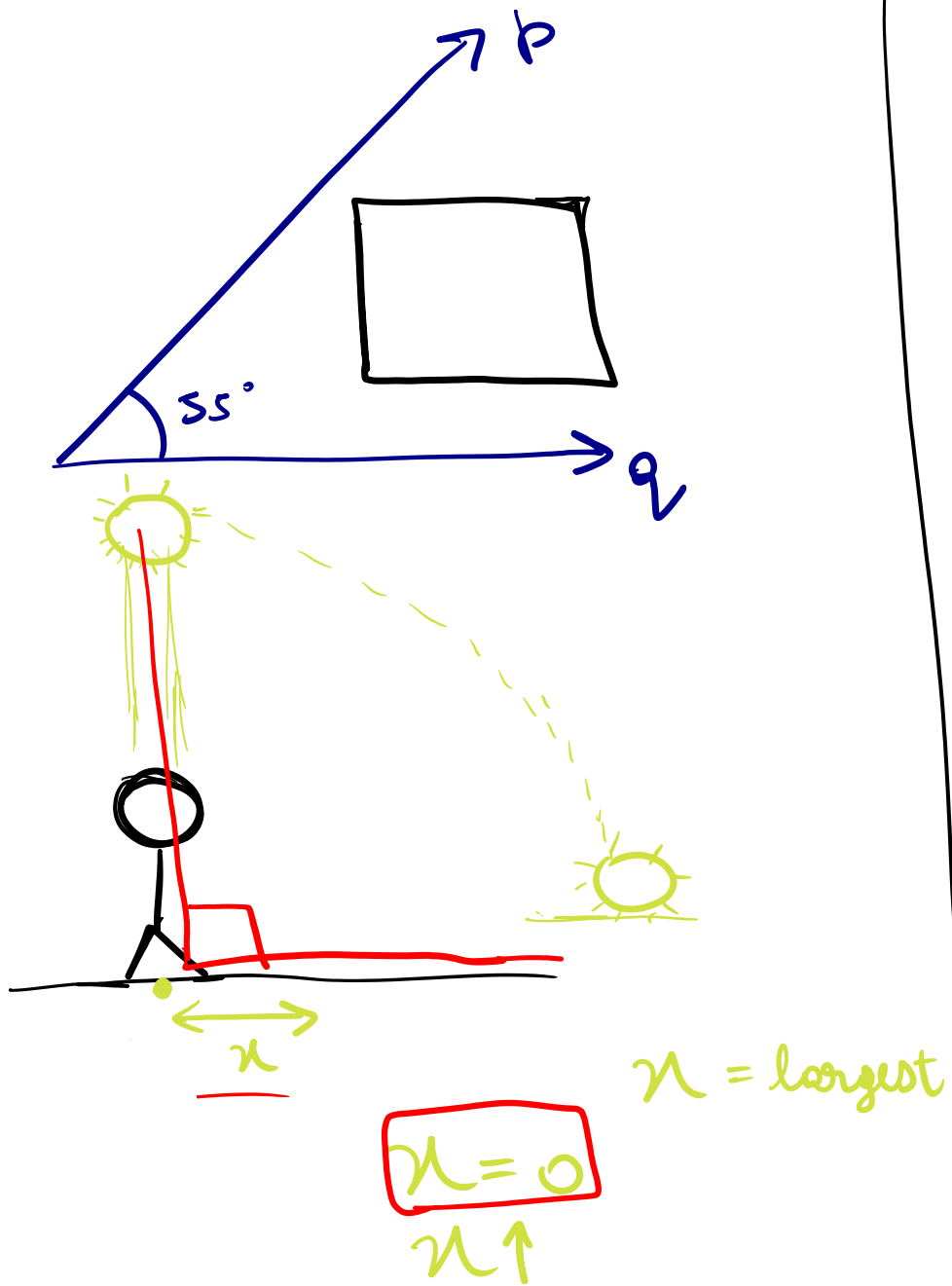
x
→
horizontal



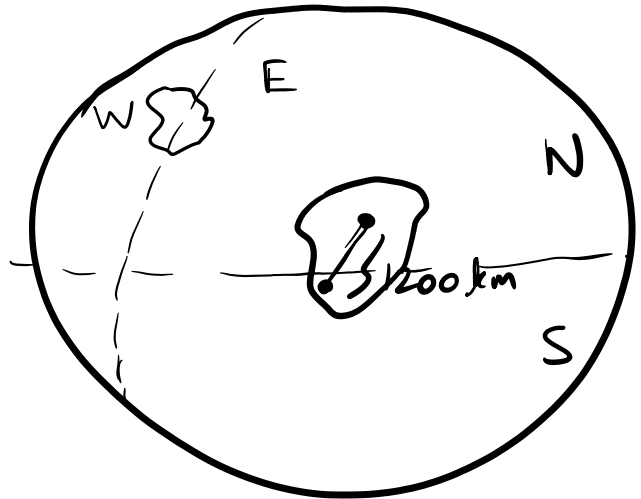
HINT

I have defined my
Coordinate system for
my own case

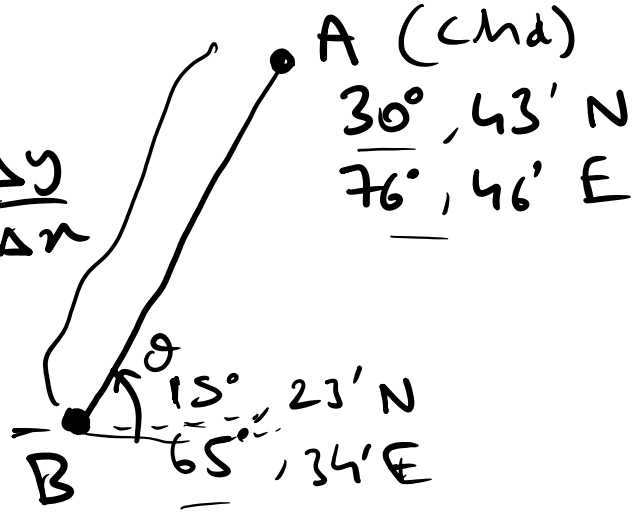
- Q - a) Why don't I use p-q system as my new set of coordinate axes?
- b) Why are the axes \perp to each other?

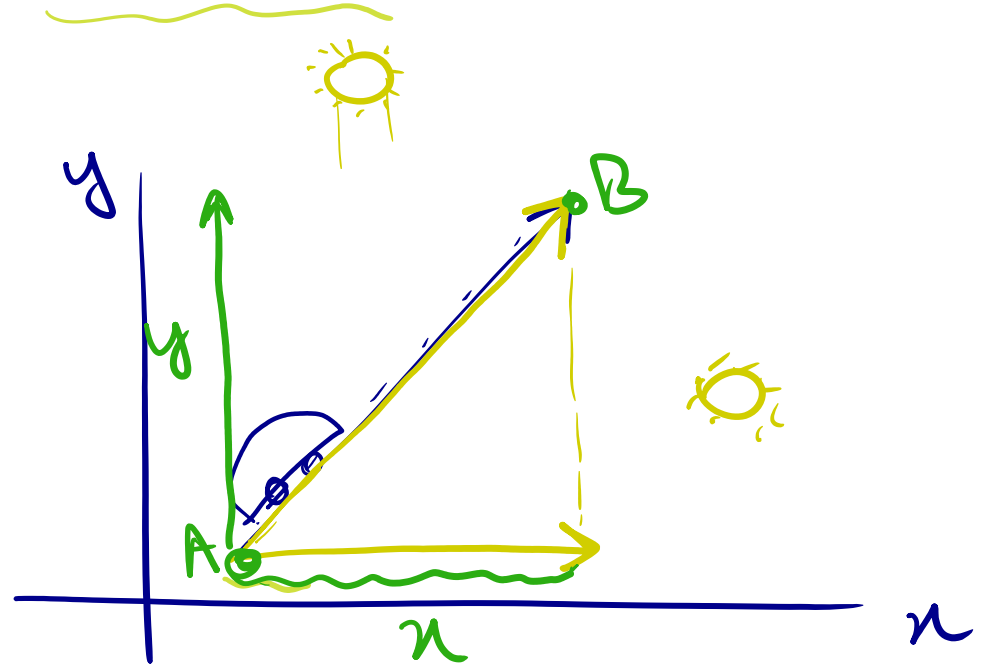
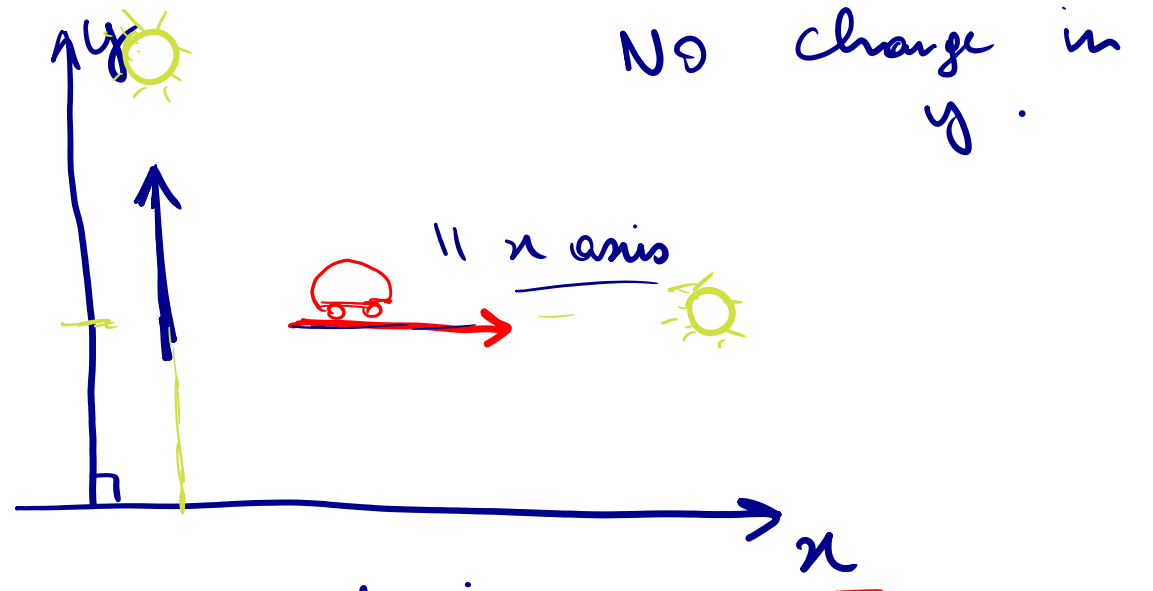
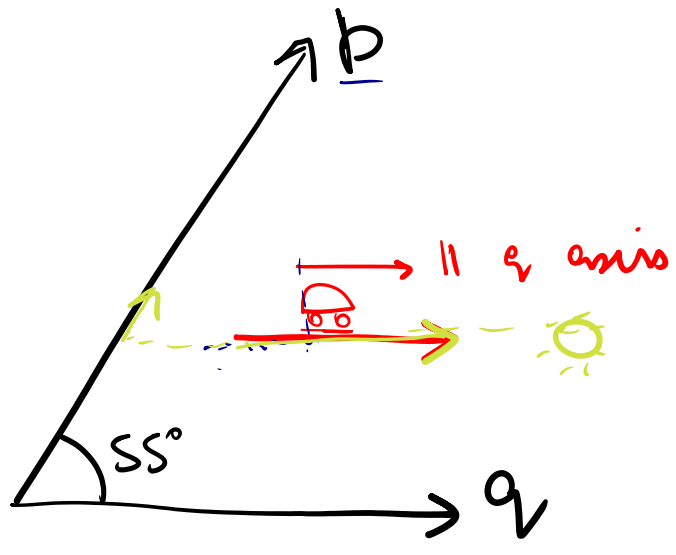


$\text{slope} = \frac{\Delta y}{\Delta x}$

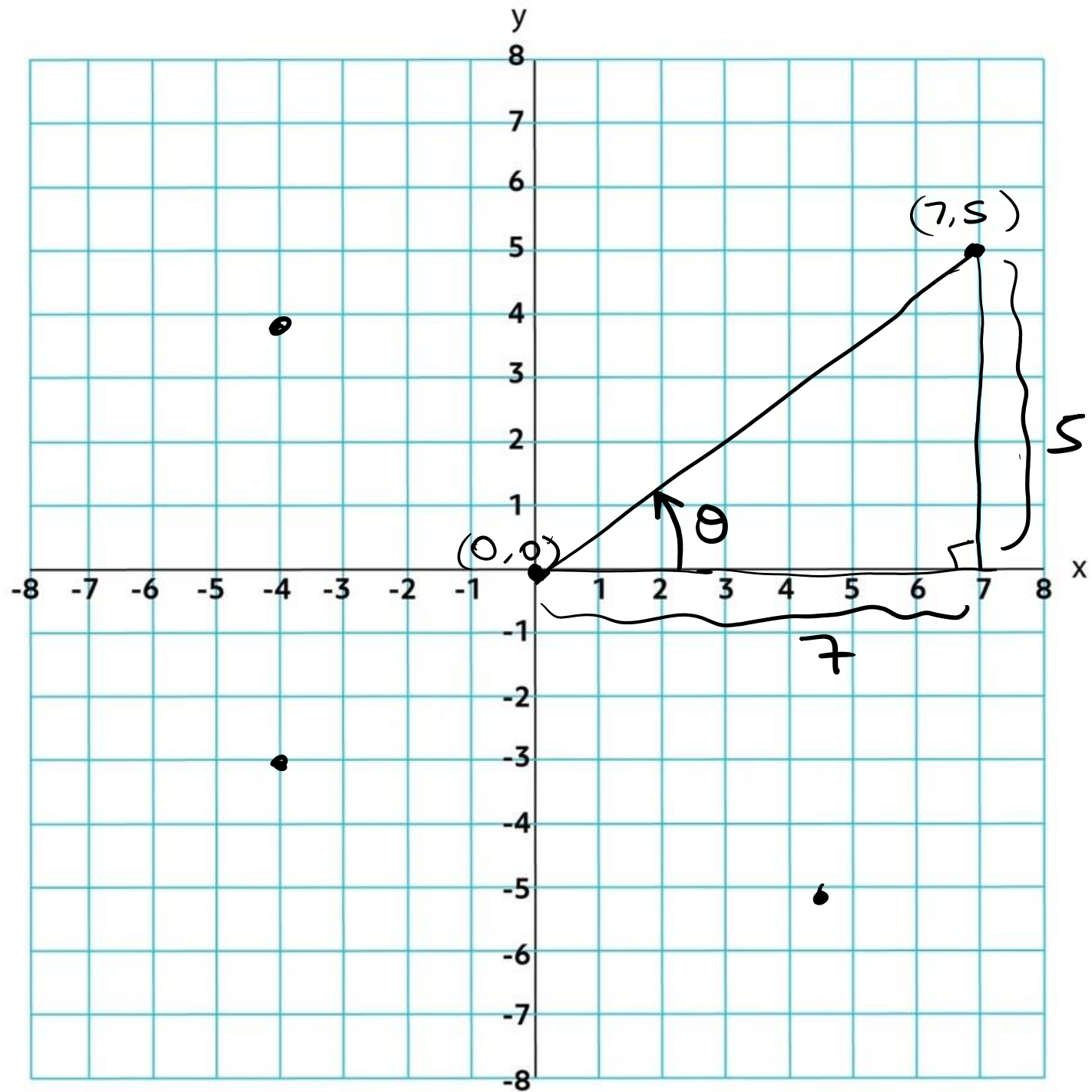


NS
 EW

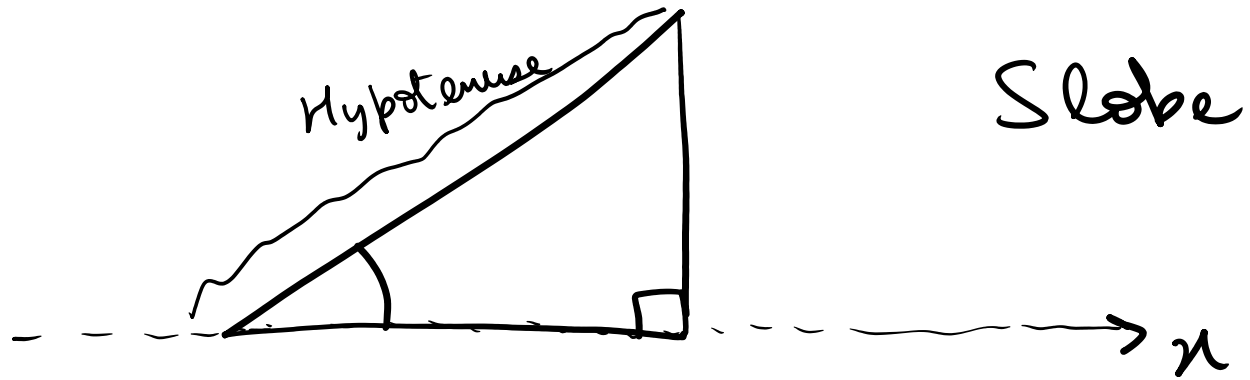


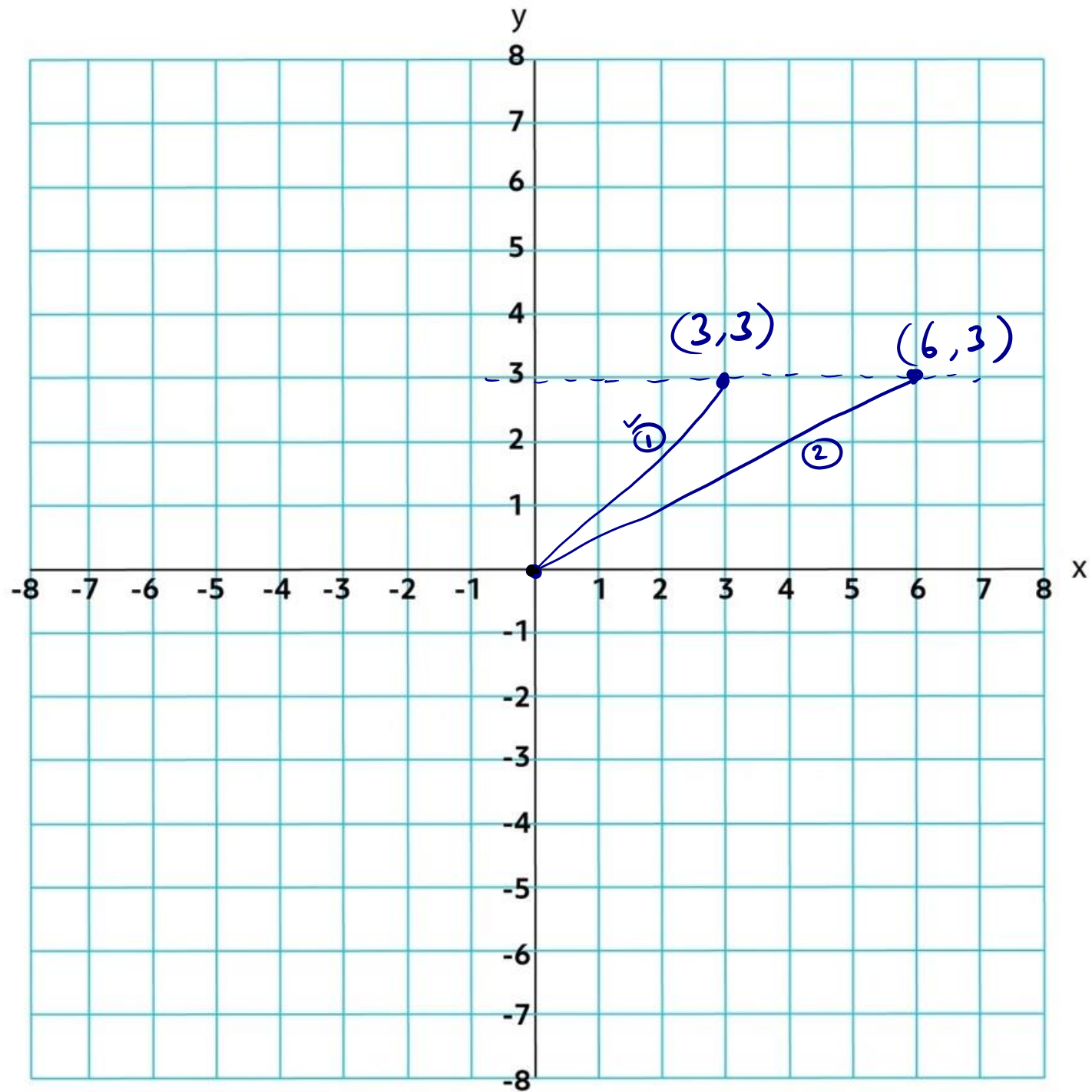


|| Cartesian System

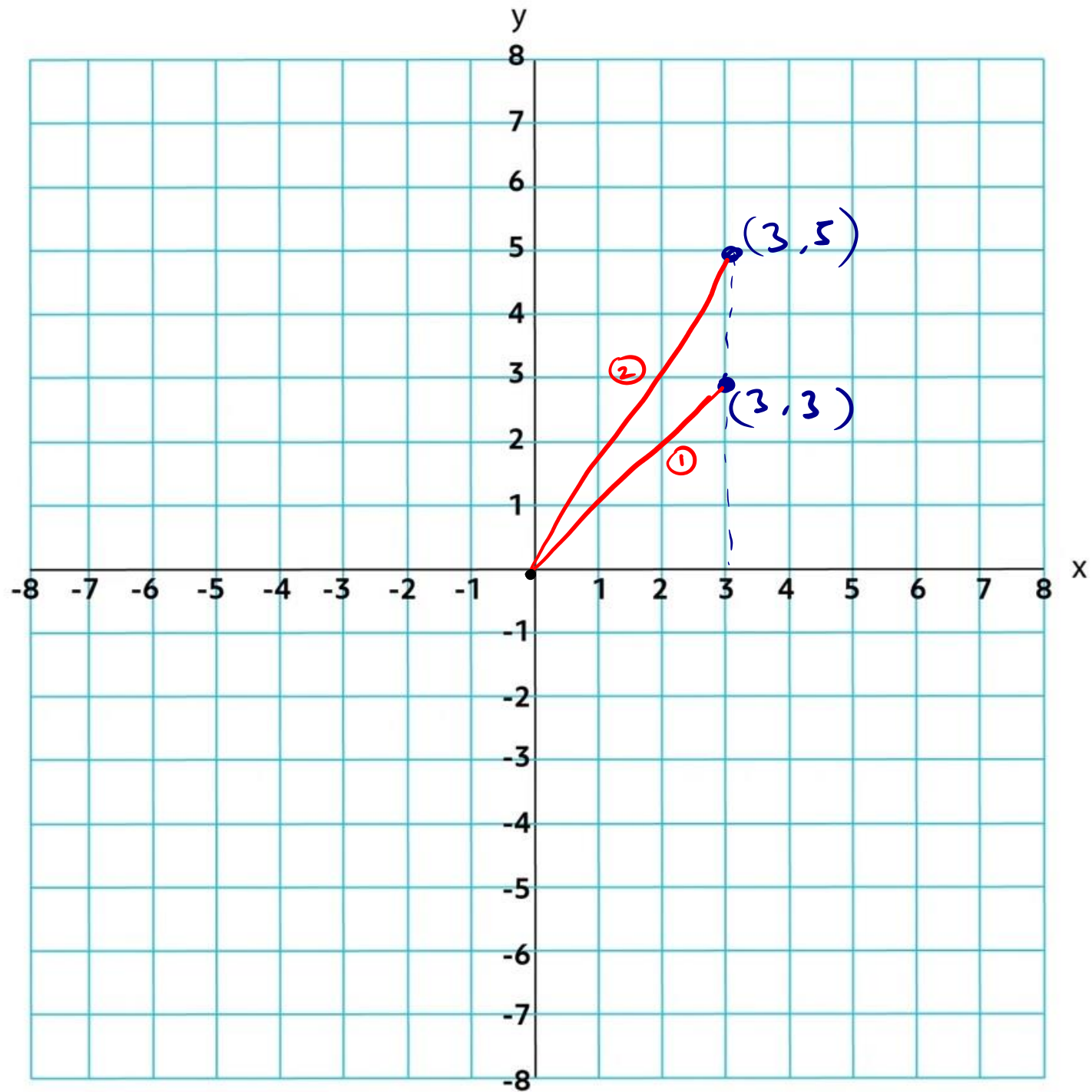


$$\sin \theta = \frac{5}{\sqrt{60}}$$
$$\cos \theta = \frac{7}{\sqrt{60}}$$
$$\tan \theta = \frac{5}{7}$$





For same
 y ,
lessen the
 x coordinate,
More is the
Slope ✓



For same x ,
more the value
of y .
more is
the slope

$$\frac{x}{y} \uparrow, \quad \frac{y}{x} \uparrow$$

Slope \downarrow
slope \uparrow

$$\left. \begin{array}{l} \text{Slope} \propto y \\ \text{Slope} \propto \frac{1}{x} \end{array} \right\}$$

$$\text{Slope} \propto \frac{y}{x}$$

$$B_y = 6$$

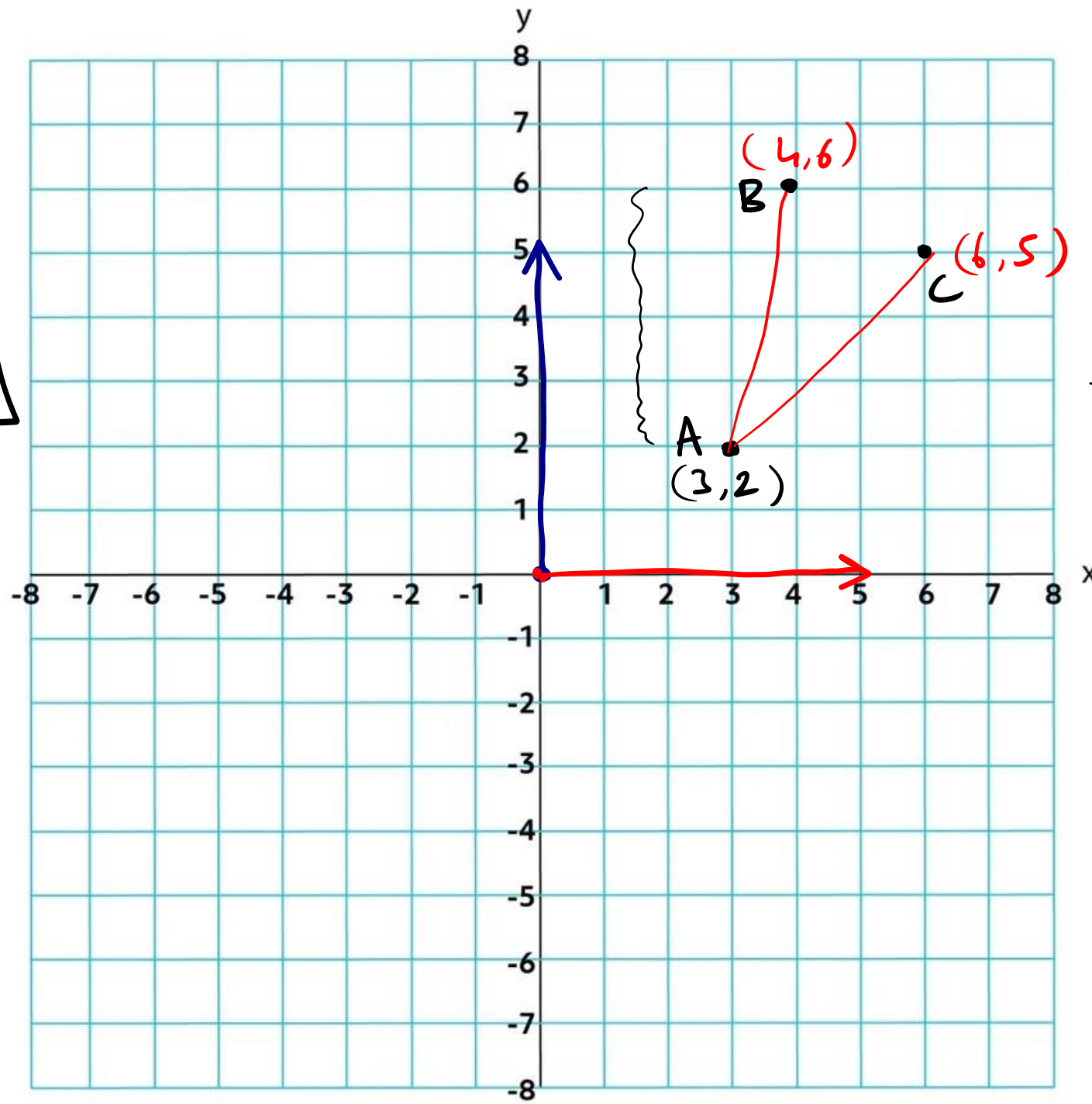
$$A_y = 2$$

$$B_y - A_y = \boxed{4}$$

$$C_y = 5$$

$$A_y = 2$$

$$C_y - A_y = \boxed{3}$$



$$B_x = 4$$

$$A_x = 3$$

$$B_x - A_x = \boxed{1}$$

$$C_x = 6$$

$$A_x = 3$$

$$C_x - A_x = 3$$

$$\Delta y = 5$$

$$\Delta x = 0$$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{5}{0}$$

$$\uparrow \boxed{0, \infty}$$

$$\text{Slope}_{BA} = \frac{\Delta y}{\Delta x} = \frac{4}{1} = 4$$

$$\text{Slope}_{CA} = \frac{\Delta y}{\Delta x} = \frac{3}{3} = 1$$

$$\frac{5}{1} = 5$$

$$\frac{5}{0.5} = 10$$

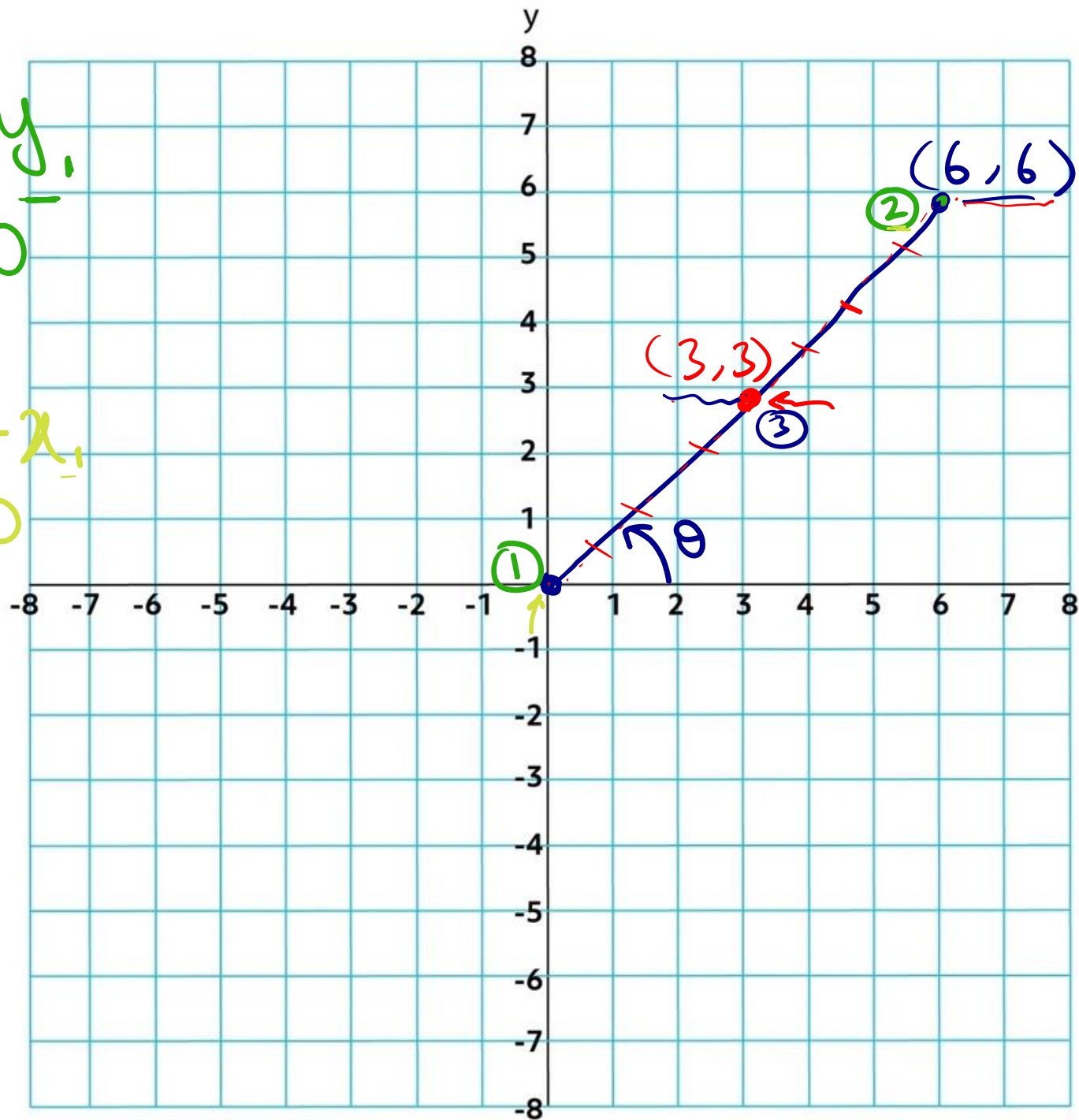
$$\frac{5}{0.05} = 100$$

$$\frac{5}{0.0005} = 10^6$$

$$\begin{aligned}\Delta y &= \underline{y_2} - \underline{y_1} \\ &= 6 - 0 \\ &= 6\end{aligned}$$

$$\begin{aligned}\Delta x &= \underline{x_2} - \underline{x_1} \\ &= 6 - 0 \\ &= 6\end{aligned}$$

$$\begin{aligned}\text{Slope} &= \frac{\Delta y}{\Delta x} \\ &= \frac{6}{6} \\ &= 1\end{aligned}$$



1) Slope of this line?

$$\underline{y = x}$$

$$\begin{aligned}\Delta y &= 6 - 3 = 3 \\ \Delta x &= 6 - 3 = 3\end{aligned}$$

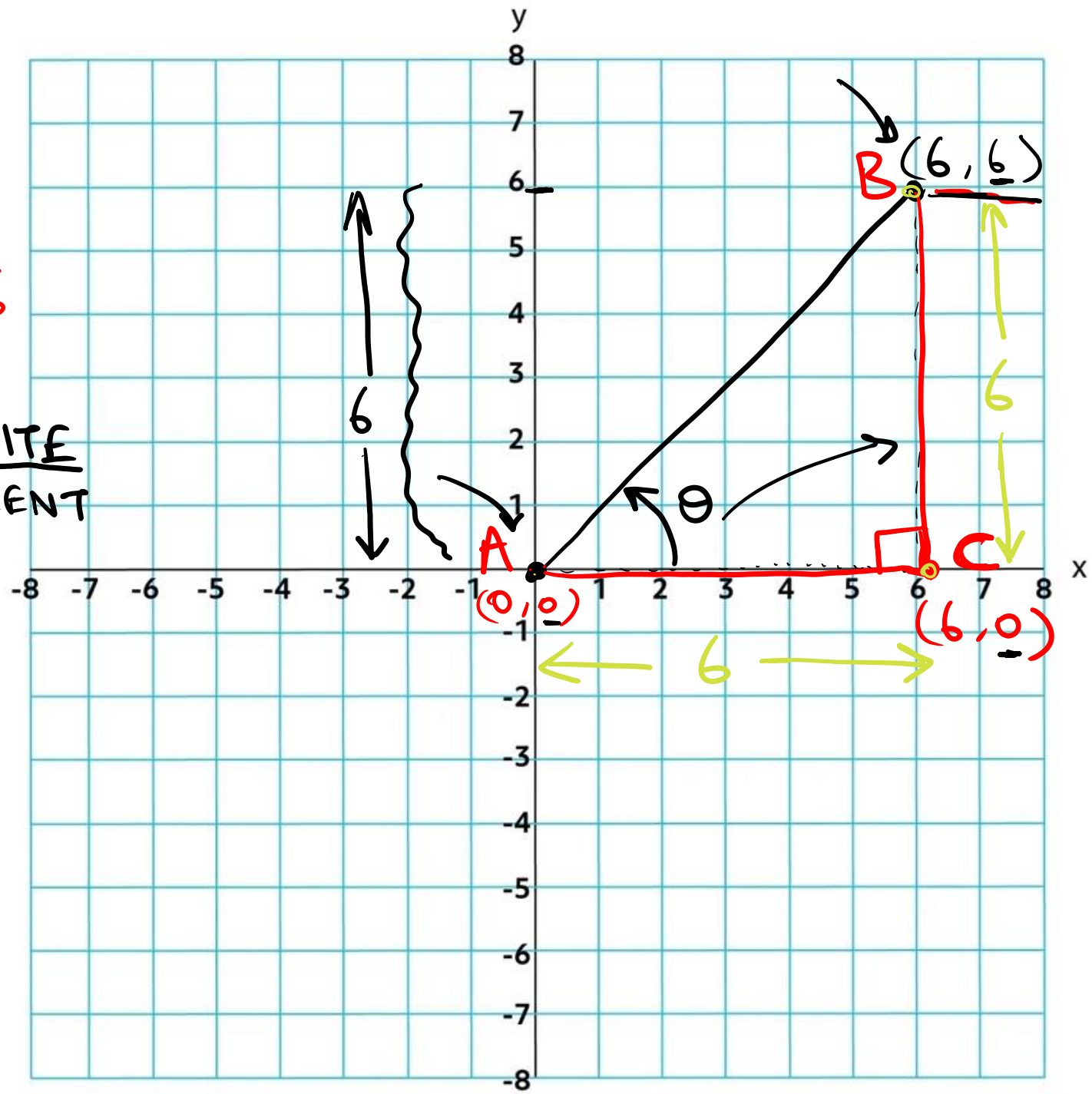
$$\begin{aligned}3 - 2 \\ \Delta y &= 3 - 6 = -3 \\ \Delta x &= 3 - 6 = -3 \\ \underline{\text{Slope}} &= 1\end{aligned}$$

Slope = $\Delta y / \Delta x$

Slope AB = $\frac{6-0}{6-0}$
= 1

$\frac{\Delta y}{\Delta x} = \frac{\text{OPPOSITE}}{\text{ADJACENT}}$

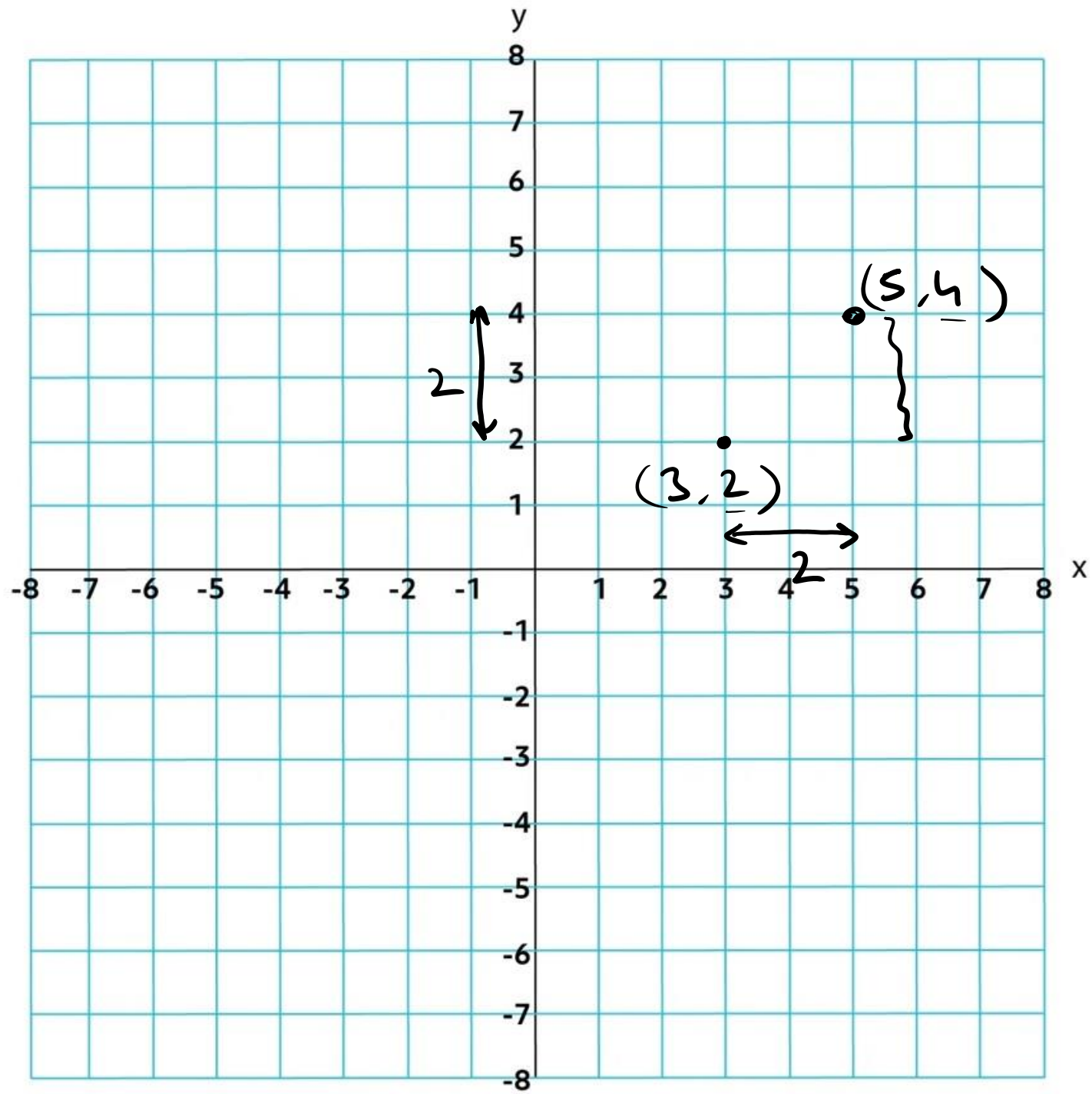
= $\tan \theta$



$\theta = 0^\circ$
Slope = 0

$\theta = 90^\circ$
Slope $\rightarrow \infty$

Hypotenuse
= $\sqrt{(6)^2 + (6)^2}$
= $6\sqrt{2}$



$$4 - 2 = 2$$
$$5 - 3 = 2$$

1) Slope of the line joining AB?

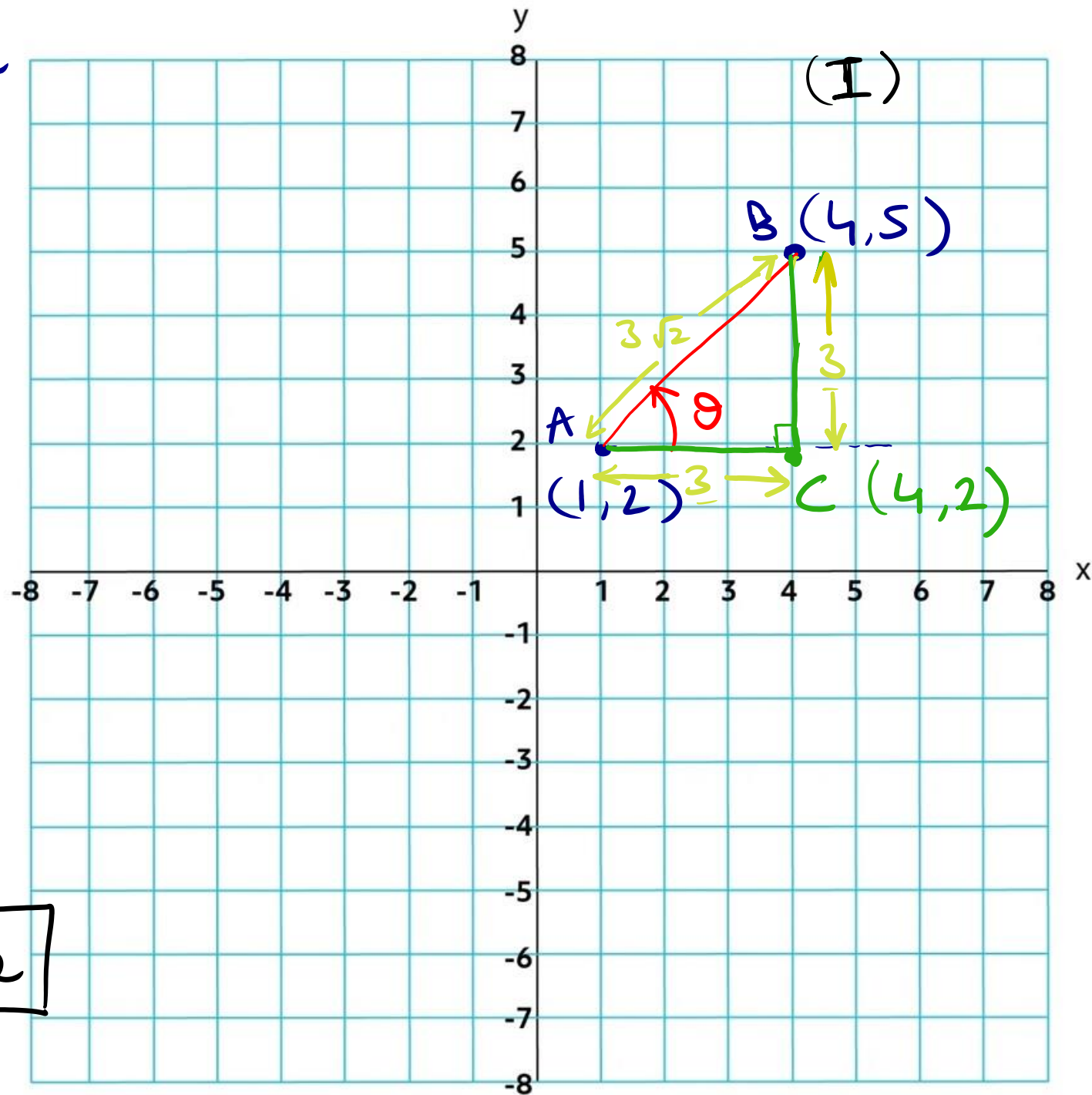
2) $\theta = ?$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

3) $\tan \theta = ?$

$$\begin{aligned} \text{slope} &= \frac{3}{3} \\ &= 1 \checkmark \end{aligned}$$

$$\boxed{\tan \theta = \text{slope}}$$



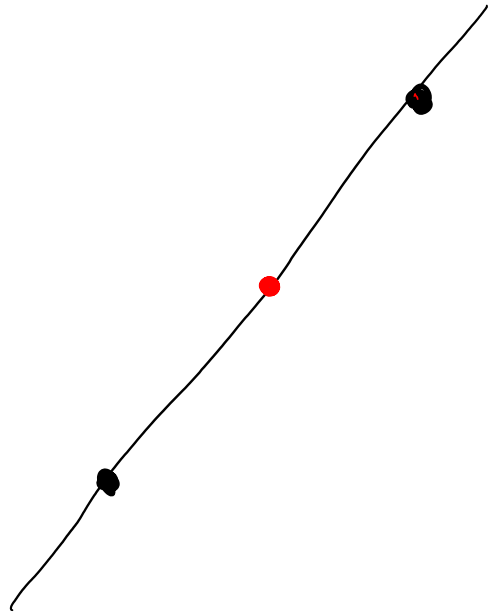
$$\begin{aligned} &\sqrt{3^2 + 3^2} \\ &\sqrt{2 \times 3^2} \\ &\quad \uparrow \\ &\boxed{3\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \underline{\sin \theta} &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{3}{3\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \underline{\sin 45^\circ} &= \frac{1}{\sqrt{2}} \\ \theta &= 45^\circ \end{aligned}$$

$$\underline{\tan \theta = 1 \checkmark}$$

Line \rightarrow If I join any 2 points,
I get a unique line



Q- { 3rd point has to be on the
line joining these points. }

HOMEWORK

Q 1) What is the value of $\tan 0^\circ$ and $\tan 90^\circ$?
(HINT - Use the definition of the slope of the line)

Q 2) What is $\sin 90^\circ$ and $\sin 0^\circ$?

Q 3) What is $\cos 90^\circ$ and $\cos 0^\circ$?

Q 4) a) What is $\tan 45^\circ$?

b) Find $\sin 45^\circ$ and $\cos 45^\circ$. Why are they equal?

HINT - Construct a Δ on the Cartesian system to approach problems 2-4.

Q5) Find the slopes of the following lines

a) AO

b) BO

c) CO

d) DO

e) FO

